Introduction

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1 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [16]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15].

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended to the interested reader.

2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_+ \times CP_2$, where M^4_+ denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [17, 18, 19, 20]. The identification of the space-time as a submanifold [21, 22] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity [Misner-Thorne-Wheeler, Logunov *et al*]. The actual choice $H = M^4_+ \times CP_2$ implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universies" of GRT) and the nonconservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [23, 24, 25]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [10, 8, 9, 13, 11, 12, 14, 15].

3.3.1 Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \ ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

3.3.2 The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij} \rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

3.3.3 Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m-M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

3.3.4 Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [I1]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

3.3.5 Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

3.3.6 p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [E1]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 identifiable as space of hyper-octonions or as surfaces in $M^4 \times CP_2$ [E2].

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

3.5.1 Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale [35] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [D6].

Already before learning about Nottale's paper I had proposed the possibility that Planck constant is quantized [E9] and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [D6].

3.5.2 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

3.5.3 p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

a) Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

b) The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [M3].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [M3]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [C6]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [C6]. It must be however emphasized that the relation of this picture to the model of quantized gravitational Planck constant h_{gr} appearing in Nottale's model is not yet completely understood.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$. Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4\ CP_2$ times so that $2^{127}-1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k=127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [L1]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p. By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p=2$-adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by $\hbar/\hbar_0$. One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [L2].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k=4$ level of hierarchy and a time scale of .1 seconds [J6], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [M3]. $k = 7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k = 7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k = 5$ would correspond to time scale of short term memories measured in minutes and $k = 6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [L2, M3]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

4 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [16]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15].

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended to the interested reader.

5 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

5.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_+ \times CP_2$, where M^4_+ denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [17, 18, 19, 20]. The identification of the space-time as a sub-manifold [21, 22] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity [Misner-Thorne-Wheeler, Logunov *et al*]. The actual choice $H = M^4_+ \times CP_2$ implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

5.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

5.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universies" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

6 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

6.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [23, 24, 25]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

6.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces?

Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

6.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [10, 8, 9, 13, 11, 12, 14, 15].

6.3.1 Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \ ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

6.3.2 The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij} \rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

6.3.3 Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m-M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-

surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

6.3.4 Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [I1]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

6.3.5 Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

6.3.6 p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [E1]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

6.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 identifiable as space of hyper-octonions or as surfaces in $M^4 \times CP_2$ [E2].

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

6.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

6.5.1 Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

The earlier work with topological quantum computation [E9] had already led to the idea that Planck constant could depend on the quantum phase $q = exp(i\pi/n)$. The first attempts to understand the large values of the Planck constant led to a badly wrong formula for this dependence. The improved understanding of Jones inclusions and their role in TGD [C6] allowed to deduce an extremely simple formula for the Planck constant, as a matter for the two separate Planck constants assignable to with M^4 and CP_2 degrees of freedom appearing as scaling factors of the corresponding metrics. These Planck constants are given by the formulas $\hbar(M^4) = n(CP_2)\hbar_0$ and $\hbar(CP_2) = n(M^4)\hbar_0$ in terms of integers defining the corresponding quantum phases. The far reaching implication is that Planck constants can have arbitrarily large values.

Furthermore, the values of n for which the quantum phase is expressible using only iterated square root operation (corresponding polygon is obtained by ruler and compass construction) are of special interest since they correspond to the lowest evolutionary levels for cognition so that corresponding systems should be especially abundant in the Universe. It should be noticed that this quantization does not depend at all on the parameter v_0 appearing in the formula of Nottale and this gives strong additional constraints to the ratios of planetary masses and also on the masses themselves if one assumes that the gravitational Planck constant corresponds to the values allowed by ruler and compass construction.

The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character. This would apply to QCD and to atoms with $Z > 137$ and to any other system.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [D6].

Further predictions follow from the observation that the quantum phases expressible using only an extension of rationals involving iterated square root operation are in a preferred position p-adically. These phases correspond to n-polygons constructible using only ruler and compass. The corresponding values of n are expressible as a product of a power of 2 and of Fermat primes each one appearing at most once. This fixes the number theoretic anatomy of $\hbar_{gr}$ in turn posing strong constraints on planetary masses. The predictions for ratios are satisfied with a better than 10 per cent accuracy. The value of v_0 deduced from the $\hbar_{gr} = GMm/v_0$ for Sun-Earth system gives $v_0 = 2^{-11}$ favored also by the quantum model of living matter whereas the empirical value is consistent with $v_0 = 1/(2^7 \times 17)$. The discrepancy can be understood by noticing that only dark matter of Earth contributes to $\hbar_{gr} = GMm/v_0$. Denoting the fraction of dark matter by $1/(1+\epsilon)$ one finds $\epsilon \simeq 6$ per cent for the fraction of the visible matter: the official value in cosmic length scales is 4 per cent.

6.5.2 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

6.5.3 p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

a) Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

b) The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [M3].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [M3]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and simple physical arguments imply a general formula for the allowed values of the scaling factor λ of Planck constant as the integer n appearing in the quantum phase $q = exp(i\pi/n)$ associated with the Jones inclusion in question [C6]. The consistency with the formula for the gravitational Planck constant h_{gr} appearing in Nottale's model poses very strong constraints on the ratios of planetary masses and implies $v_0 = 1/\lambda = 2^{-11}$ as an exact formula.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$. Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4\ CP_2$ times so that $2^{127}-1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k=127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [L1]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p. By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p = 2$-adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by $\hbar/\hbar_0$. One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [L2].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k = 4$ level of hierarchy and a time scale of .1 seconds [J6], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [M3]. $k = 7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k = 7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k = 5$ would correspond to time scale of short term memories measured in minutes and $k = 6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [L2, M3]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

7 Bird's eye of view about the topics of the book

The topics of this book are following.

a) In the first part of the book I will try to give an overall view about TGD and the evolution of TGD to its recent form. I cannot avoid the use of various concepts without detailed definitions and my hope is that reader only gets a bird's eye of view about TGD. Two visions about physics are discussed. According to the first vision physical states of the Universe correspond to classical spinor fields in the world of the classical worlds identified as 3-surfaces or equivalently as corresponding 4-surfaces analogous to Bohr orbits and identified as special extreme of Kähler action. TGD as a generalized number theory vision leading naturally also to the emergence of p-adic physics as physics of cognitive representations is the second vision. Number theoretical vision involves three loosely related approaches: fusion of real and various p-adic physics to a larger whole as algebraic continuations of what might be called rational physics; space-time as a hyper-quaternionic surface of hyper-octonion space, and space-time surfaces as a representations of infinite primes.

b) The second part of the book is devoted to the vision about physics as infinite-dimensional configuration space geometry. The basic idea is that classical spinor fields in infinite-dimensional "world of classical worlds", space of 3-surfaces in $M^4 \times CP_2$ describe the quantum states of the Universe. Quantum jump remains the only purely quantal aspect of quantum theory in this approach since there is no quantization at the level of the configuration space. Space-time surfaces correspond to special extremals of the Kähler action analogous to Bohr orbits and define what might be called classical TGD discussed in the first chapter. The construction of the configuration space geometry and spinor structure are discussed in remaining chapters.

c) The third part of the book carries title "Algebraic Physics". Classical physics is an exact part of the configuration space geometry and the identification of space-time surfaces as generalized Bohr orbits is not a mere approximation. Path integral is replaced by a functional integral over 3-surfaces and there are reasons to expect that the decomposition of the configuration space to a union of symmetric spaces allows to carry out the functional integral exactly. This together with quantum classical correspondence and Bohr orbit property suggests a generalization of the duality symmetry of hadron models stating that the addition of loops does not affect the physical amplitude assignable to a given diagram. Hopf algebras are used to formulate this symmetry. In the second chapter the profound implications of the fact that configuration space spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 are considered.

d) The physical applications of TGD are the topic of the fourth part of the book. The cosmological and astrophysical applications of the many-sheeted space-time are summarized and the applications to elementary particle physics are discussed at the general level. TGD explains particle families in terms of generation genus correspondences (particle families correspond to 2-dimensional topologies labelled by genus). The notion of elementary particle vacuum functional is developed leading to an argument that the number of light particle families is three is discussed. The general theory for particle massivation based on p-adic thermodynamics is discussed at the general level. The detailed calculations of elementary particle masses are not however carried out and can be found at [F3, F4, F5].

The seven online books about TGD [1, 2, 4, 5, 3, 6, 7] and eight online books about TGD inspired theory of consciousness and quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended for the reader willing to get overall view about what is involved.

8 The contents of the book

8.1 PART I: General Overview

8.1.1 Overall View About Evolution of TGD

This chapter provides a bird's eye view about TGD in its 27^{th} birthday with the hope that this kind of summary might make it easier to follow the more technical representation provided by sub-sequent chapters. The geometrization of fundamental interactions assuming that space-times are representable as 4-surfaces of $H = M^4_+ \times CP_2$ is wherefrom everything began. The two manners to understand TGD is TGD as a Poincare invariant theory of gravitation obtained by fusing special and general relativities, and TGD as a generalization of string model obtained my replacing 1-dimensional strings with 3-surfaces. The fusion of these approaches leads to the notion of the many-sheeted space-time.

The evolution of quantum TGD involve five threads which have become more and more entangled with each other. The first great vision was the reduction of the entire quantum physics (apart from quantum jump) to the geometry of classical spinor fields of the infinite-dimensional space of 3-surfaces in H, the great idea being that infinite-dimensional Kähler geometric existence and thus physics is unique from the requirement that it is free of infinities. The outcome is geometrization and generalization of the known structures of the quantum field theory and of string models.

The second thread is p-adic physics. p-Adic physics was initiated by more or less accidental observations about reduction of basic mass scale ratios to the ratios of square roots of Mersenne primes and leading to the p-adic thermodynamics explaining elementary particle mass scales and masses with an unexpected success. p-Adic physics turned eventually to be the physics of cognition and intentionality. Consciousness theory based ideas have led to a generalization of the notion of number obtained by gluing real numbers and various p-adic number fields along common rationals to a more general structure and implies that many-sheeted space-time contains also p-adic space-time sheets serving as space-time correlates of cognition and intentionality. The hypothesis that real and p-adic physics can be regarded as algebraic continuation of rational number based physics provides extremely strong constraints on the general structure of quantum TGD.

TGD inspired theory of consciousness can be seen as a generalization of quantum measurement theory replacing the notion of observer as an outsider with the notion of self. The detailed analysis of what happens in quantum jump have brought considerable understanding about the basic structure of quantum TGD itself. It seems that even quantum jump itself could be seen as a number theoretical necessity in the sense that state function reduction and state preparation by self measurements are necessary in order to reduce the generalized quantum state which is a formal superposition over components in different number fields to a state which contains only rational or finitely-extended rational entanglement identifiable as bound state entanglement. The number theoretical information measures generalizing Shannon entropy (always non-negative) are one of the important outcomes of consciousness theory combined with p-adic physics.

Physics as a generalized number theory is the fourth thread. The key idea is that the notion of divisibility could make sense also for literally infinite numbers and perhaps make them useful from the point of view of physicist. The great surprise was that the construction of infinite primes corresponds to the repeated quantization of a super-symmetric arithmetic quantum field theory. This led to the vision about physics as a generalized number theory involving infinite primes, integers, rationals and reals, as well as their quaternionic and octonionic counterparts. A further generalization is based on the generalization of the number concept already mentioned. Space-time surfaces could be regarded in this framework as concrete representations for infinite primes and integers, whereas the dimensions 8 and 4 for imbedding space and space-time surface could be seen as reflecting the dimensions of octonions and quaternions and their hyper counterparts obtained by multiplying imaginary units by $\sqrt{-1}$. Also the dimension 2 emerges naturally as the maximal dimension of commutative sub-number field and relates to the ordinary conformal invariance central also for string models.

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

This chapter represents a overall view about evolution of classical TGD and of p-adic concepts, a summary of the ideas generated by TGD inspired theory of consciousness, the vision about physics as a generalized number theory.

8.1.2 Overall View About Quantum TGD

This chapter provides a summary about quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis this vision.

1. Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years.

a) The basic vision is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom. The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

b) Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This has led to a profound understanding of quantum TGD. The outcome of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

c) p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only superconformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

d) The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. This leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. This leads to an explicit formula for the Dirac determinant. What is remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of partonic 2-surfaces: they should correspond to preferred extremals of Kähler action. Thus hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality.

e) HFFs lead also to an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, CP_2 could be interpreted as a structure related to octonions. This would mean that TGD could be seen also as a generalized number theory.

2. Ideas related to the construction of S-matrix

The construction of S-matrix involves several ideas that have emerged during last years.

a) Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. S-matrix and density matrix are unified to the notion of M-matrix expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

b) The notion of measurement resolution represented in terms of inclusions of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This algebra effectively replaces complex numbers as coefficient fields and the condition that its action commutes with the M-matrix implies that M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this fixes possible M-matrices to a very high degree.

c) Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory.

8.1.3 TGD and M-Theory

In this chapter a critical comparison of M-theory and TGD as two competing theories is carried out. Dualities and black hole physics are regarded as basic victories of M-theory.

a) The counterpart of electric magnetic duality plays an important role also in TGD and it has become clear that it might change the sign of Kähler coupling strength rather than leaving it invariant. The different signs would be related to different time orientations of the space-time sheets. This option is favored also by TGD inspired cosmology.

b) The AdS/CFT duality of Maldacena involved with the quantum gravitational holography has a direct counterpart in TGD with 3-dimensional causal determinants serving as holograms so that the construction of absolute minima of Kähler action reduces to a local problem.

c) The duality of string models relating Kaluza-Klein quantum numbers with YM quantum numbers could generalize to a duality between 7-dimensional light like causal determinants of the imbedding space (analogs of "big bang") and 3-dimensional light like causal determinants of space-time surface (analogs of black hole horizons).

In TGD framework black holes are possible but putting black holes and particles in the same basket seems to be mixing of apples with oranges. The role of black hole horizons is taken in TGD by 3-D light like causal determinants, which are much more general objects. Black hole-elementary particle correspondence and p-adic length scale hypothesis have already earlier led to a formula for the entropy associated with elementary particle horizon.

The discussion of the basic weaknesses of M-theory is motivated by the fact that the few predictions of the theory are wrong which has led to the introduction of anthropic principle to save the theory. The mouse as a tailor history of M-theory, the lack of a precise problem to which M-theory would be a solution, the hard nosed reductionism, and the censorship in Los Alamos archives preventing the interaction with competing theories could be seen as the basic reasons for the recent blind alley in M-theory.

8.2 PART II: Physics as infinite-dimensional spinor geometry in the world of classical worlds

8.2.1 Classical TGD

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable.

1. General considerations

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all extremals or at least the absolute minima of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Kähler current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

2. Absolute minimization of Kähler action and second law of thermodynamics

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that absolute minima of Kähler action correspond to space-time sheets which asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Absolute minimization abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. Also the equivalence of absolute minimization with the second law strongly suggests itself. Of course, one must keep mind open for the possibility that it is the second law of thermodynamics which replaces absolute minimization as the fundamental principle.

3. The dimension of CP_2 projection as classifier for the fundamental phases of matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. This phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase is the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

4. Specific extremals of Kähler action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

a) The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

b) The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynmann diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

c) There are also various non-vacuum extremals.

i) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

ii) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

iii) In the so called Maxwell's phase, ordinary Maxwell equations for the induced Kähler field are satisfied in an excellent approximation. A special case is provided by a radially symmetric extremal having an interpretation as the space-time exterior to a topologically condensed particle. The sign of the gravitational mass correlates with that of the Kähler charge and one can understand the generation of the matter antimatter asymmetry from the basic properties of this extremal. The possibility to understand the generation of the matter antimatter asymmetry directly from the basic equations of the theory gives strong support in favor of TGD in comparison to the ordinary EYM theories, where the generation of the matter antimatter asymmetry is still poorly understood.

8.2.2 Construction of configuration space geometry

The topics of this chapter are the purely geometric aspects of the vision about physics as an infinite-dimensional Kähler geometry of the "world of classical worlds", with " classical world" identified either as 3-D surface of the unique Bohr orbit like 4-surface traversing through it. The non-determinism of Kähler action forces to generalize the notion of 3-surfaces so that unions of space-like surfaces with time like separations must be allowed. The considerations are restricted mostly to real context and the problems related to the p-adicization are discussed later.

There are two separate tasks involved.

a) Provide configuration space of 3-surfaces with Kähler geometry which is consistent with 4-dimensional general coordinate invariance so that the metric is Diff^4 degenerate. General coordinate invariance implies that the definition of metric must assign to a give 3-surface X^3 a 4-surface as a kind of Bohr orbit $X^4(X^3)$.

b) Provide the configuration space with a spinor structure. The great idea is to identify configuration space gamma matrices in terms of super algebra generators expressible using second quantized fermionic oscillator operators for induced free spinor fields at the space-time surface assignable to a given 3-surface. The isometry generators and contractions of Killing vectors with gamma matrices would thus form a generalization of Super Kac-Moody algebra.

From the experience with loop spaces one can expect that there is no hope about existence of well-defined Riemann connection unless this space is union of infinite-dimensional symmetric spaces with constant curvature metric and simple considerations requires that Einstein equations are satisfied by each component in the union. The coordinates labelling these symmetric spaces are zero modes having interpretation as genuinely classical variables which do not quantum fluctuate since they do not contribute to the line element of the configuration space.

The construction of the Kähler structure involves also the identification of complex structure. Direct construction of Kähler function as action associated with a preferred Bohr orbit like extremal for some physically motivated action action leads to a unique result. Second approach is group theoretical and is based on a direct guess of isometries of the infinite-dimensional symmetric space formed by 3-surfaces with fixed values of zero modes. The group of isometries is generalization of Kac-Moody group obtained by replacing finite-dimensional Lie group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone.

8.2.3 Configuration space spinor structure

Quantum TGD should be reducible to the classical spinor geometry of the configuration space. In particular, physical states should correspond to the modes of the configuration space spinor fields. The immediate consequence is that configuration space spinor fields cannot, as one might naively expect, be carriers of a definite spin and unit fermion number. Concerning the construction of the configuration space spinor structure there are some important clues.

1. Geometrization of fermionic statistics in terms of configuration space spinor structure

The great vision has been that the second quantization of the induced spinor fields can be understood geometrically in terms of the configuration space spinor structure in the sense that the anti-commutation relations for configuration space gamma matrices require anti-commutation relations for the oscillator operators for free second quantized induced spinor fields.

a) One must identify the counterparts of second quantized fermion fields as objects closely related to the configuration space spinor structure. Ramond model has as its basic field the anti-commuting field $\Gamma^k(x)$, whose Fourier components are analogous to the gamma matrices of the configuration space and which behaves like a spin $3/2$ fermionic field rather than a vector field. This suggests that the complexified gamma matrices of the configuration space are analogous to spin $3/2$ fields and therefore expressible in terms of the fermionic oscillator operators so that their anti-commutativity naturally derives from the anti-commutativity of the fermionic oscillator operators.

As a consequence, configuration space spinor fields can have arbitrary fermion number and there would be hopes of describing the whole physics in terms of configuration space spinor field. Clearly, fermionic oscillator operators would act in degrees of freedom analogous to the spin degrees of freedom of the ordinary spinor and bosonic oscillator operators would act in degrees of freedom analogous to the 'orbital' degrees of freedom of the ordinary spinor field.

b) The classical theory for the bosonic fields is an essential part of the configuration space geometry. It would be very nice if the classical theory for the spinor fields would be contained in the definition of the configuration space spinor structure somehow. The properties of the modified massless Dirac operator associated with the induced spinor structure are indeed very physical. The modified massless Dirac equation for the induced spinors predicts a separate conservation of baryon and lepton numbers. The differences between quarks and leptons result from the different couplings to the CP_2 Kähler potential. In fact, these properties are shared by the solutions of massless Dirac equation of the imbedding space.

c) Since TGD should have a close relationship to the ordinary quantum field theories it would be highly desirable that the second quantized free induced spinor field would somehow appear in the definition of the configuration space geometry. This is indeed true if the complexified configuration space gamma matrices are linearly related to the oscillator operators associated with the second quantized induced spinor field on the space-time surface and/or its boundaries. There is actually no deep reason forbidding the gamma matrices of the configuration space to be spin half odd-integer objects whereas in the finite-dimensional case this is not possible in general. In fact, in the finite-dimensional case the equivalence of the spinorial and vectorial vielbeins forces the spinor and vector representations of the vielbein group $SO(D)$ to have same dimension and this is possible for $D = 8$-dimensional Euclidian space only. This coincidence might explain the success of 10-dimensional super string models for which the physical degrees of freedom effectively correspond to an 8-dimensional Euclidian space.

d) It took a long time to realize that the ordinary definition of the gamma matrix algebra in terms of the anti-commutators $\{\gamma_A, \gamma_B\} = 2g_{AB}$ must in TGD context be replaced with $\{\gamma_A^\dagger, \gamma_B\} = iJ_{AB}$, where J_{AB} denotes the matrix elements of the Kähler form of the configuration space. The presence of the Hermitian conjugation is necessary because configuration space gamma matrices carry fermion number. This definition is numerically equivalent with the standard one in the complex coordinates. The realization of this delicacy is necessary in order to understand how the square of the configuration space Dirac operator comes out correctly.

e) The only possible option is that second quantized induced spinor fields are defined at 3-D light-like causal determinants associated with 4-D space-time sheet. The unique partonic dynamics is almost topological QFT defined by Chern-Simons action for the induced Kähler gauge potential and by the modified Dirac action constructed from it by requiring super-conformal symmetry. The resulting theory has all the desired super-conformal symmetries and is exactly solvable at parton level. It is 3-dimensional lightlike 3-surfaces rather than generic 3-surfaces which are the fundamental dynamical objects in this approach.

The classical dynamics of the interior of space-time surface defines a classical correlate for the partonic quantum dynamics and provides a realization of quantum measurement theory. It is determined by the vacuum functional identified as the Dirac determinant. There are good arguments suggesting that it reduces to an exponent of absolute extremum of Kähler action in each region of the space-time sheet where the Kähler action density has a definite sign.

2. Modified Dirac equation for induced classical spinor fields

The identification of the light-like partonic 3-surfaces as carriers of elementary particle quantum numbers inspired by the TGD based quantum measurement theory forces the identification of the modified Dirac action as that associated with the Chern-Simons action for the induced Kähler gauge potential. At the fundamental level TGD would be almost-topological super-conformal QFT in the sense that only the light-likeness condition for the partonic 3-surfaces would involve the induced metric. Chern-Simons dynamics would thus involve the induced metric only via the generalized eigenvalue equation for the modified Dirac operator involving the light-like normal of $X^3_l \subset X^4$. $N = 4$ super-conformal symmetry emerges as a maximal Super-Kac Moody symmetry for this option. The application of D to any generalized eigen-mode gives a zero mode and zero modes and generalized eigen-modes define a cohomology.

The interpretation of the solutions of the modified Dirac equation differs from the conventional one and is motivated by the construction of the Kähler function in terms of Dirac determinants associated with the modified Dirac action. This gives hopes of evaluating the exponent of Kähler function as a product of Dirac determinants of the partonic 3-surfaces associated with the space-time sheet without solving the field equations and using only the data provided by the light-likeness.

The solutions of the modified Dirac equation are interpreted as generators of exact $N = 4$ super-conformal symmetries in both quark and lepton sectors. These super-symmetries correspond to pure super gauge transformations and no spartners of ordinary particles are predicted: in particular $N = 2$ space-time super-symmetry is generated by the righthanded neutrino is absent contrary to the earlier beliefs. There is no need to emphasize the experimental implications of this finding. The original conjecture was that the eigenvalues correspond to Riemann Zeta. Zeta is however naturally replaced with that defined by the eigenvalues of the modified Dirac operator.

An essential difference with respect to standard super-conformal symmetries is that Majorana condition is not satisfied and the usual super-space formalism does not apply. Chern-Simons action however leads to an elegant mechanism allowing to obtain modified Dirac action from the super-symmetrized Chern-Simons action by integrating over Grassmann parameters using a modification of the usual Grassman integration measure.

Configuration space gamma matrices identified as super generators of super-canonical or super Kac-Moody algebras (depending on CH coordinates used) are expressible in terms of the oscillator operators associated with the eigen modes of the modified Dirac operator. Super-canonical and super Kac-Moody charges are expressible as integrals over 2-dimensional partonic surfaces X^2 and interior degrees of freedom of X^4 can be regarded as zero modes representing classical variables in one-one correspondence with quantal degrees of freedom at X^3_l as indeed required by quantum measurement theory. In fact, for certain subalgebra of super-canonical transformations the charge densities correspond to closed 2-forms expressible as 1-dimensional integrals so that stringy picture results and leads to anti-commutation relations for the induced spinor fiels consistent with those of conformal field theory. The resulting situation is highly reminiscent of WZW model and the results imply that at technical level the methods of 2-D conformal field theories should allow to construct quantum TGD.

3. The exponent of Kähler function as Dirac determinant for the modified Dirac action

Although quantum criticality in principle predicts the possible values of Kähler coupling strength, one might hope that there exists even more fundamental approach involving no coupling constants and predicting even quantum criticality and realizing quantum gravitational holography.

a) The Dirac determinant defined by the product of Dirac determinants associated with the light-like partonic 3-surfaces X_l^3 associated with a given space-time sheet X^4 is the simplest candidate for vacuum functional identifiable as the exponent of the Kähler function. One can of course worry about the finiteness of the Dirac determinant. p-Adicization requires that the eigenvalues belong to a given algebraic extension of rationals. This restriction would imply a hierarchy of physics corresponding to different extensions and could automatically imply the finiteness and algebraic number property of the Dirac determinants if only finite number of eigenvalues would contribute. The regularization would be performed by physics itself if this were the case.

b) The generalization of the imbedding space implied by the hierarchy of Planck constants turns out to be essential for the explicit construction. Generalized imbedding space is obtained by gluing together infinite number of covering spaces and factor spaces of $M^4\backslash M^2 \times CP_2\backslash S^2_{II}$, where S^2_{II} is homologically trivial geodesic sphere of CP_2. The gluing takes place along the quantum critical manifolds $M^2 \times CP_2$, $M^4 \times S^2_{II}$, and $M^2 \times S^2_{II}$.

c) Simple consistency conditions imply that D can have only one generalized eigenvalue whose over-all scale is expected to depend on p-adic prime. λ can be deduced from the requirement that it represents geometric data about X^2 and depends on the transversal coordinates of the partonic 2-surface holomorphically. Higgs vacuum expectation is naturally proportional to $\lambda(w)$, w the point of X^2. This leads to an identification of the points of the number theoretic braid as minima of the purely geometric Higgs potential associated $\lambda(w_k)$.

d) Dirac determinant can be defined as the product of $\lambda(w_k)$ at the points of the braid divided with the product of $M^4_+\pm$ distances to the quantum critical manifold $R_+ \times S^2_{II}$. Dirac should give rise to the exponents of Kähler function and Chern-Simons action. The consistency with the vacuum degeneracy leads to an essentially unique geometric construction of $\lambda(w)$. Both Kähler function and Chern-Simons action and also super-canonical conformal weights - identified as zeros of zeta associated with $\lambda(w_k)$ at points of braid - are invariant under overall scalings of λ in accordance with renormalization group invariance. One can understand p-adic coupling constant evolution in terms of dependence of the dependence of the scaling factor of λ on p-adic prime p.

e) What is remarkable, the construction of λ automatically gives rise to a construction of 4-D space-time sheet assigned to the 3-D light-like surface and it remains to be shown that preferred extremum of Kähhler action is in question.

4. Super-conformal symmetries

The almost topological QFT property of partonic formulation based on Chern-Simons action and corresponding modified Dirac action allows a rich structure of $N = 4$ super-conformal symmetries. In particular, the generalized Kac-Moody symmetries leave corresponding X^3-local isometries respecting the light-likeness condition. A rather detailed view about various aspects of super-conformal symmetries emerge leading to identification of fermionic anti-commutation relations and explicit expressions for configuration space gamma matrices and Kähler metric. This picture is consistent with the conditions posed by p-adic mass calculations.

Number theoretical considerations play a key role and lead to the picture in which effective discretization occurs so that partonic two-surface is effectively replaced by a discrete set of algebraic points belonging to the intersection of the real partonic 2-surface and its p-adic counterpart obeying the same algebraic equations. The restriction to the minima of the purely geometric correlate of the vacuum expectation value of Higgs field defined by the eigenvalue of the modified Dirac operator selects a subset of points defining number theoretic braids. This implies effective discretization of super-conformal field theory giving N-point functions defining vertices via discrete versions of stringy formulas.

8.3 PART III: Algebraic Physics

8.3.1 Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD

The great dream of a physicist believing in reductionism and TGD would be a formalism generalizing Feynman diagrams allowing any graduate student to compute the predictions of the theory. TGD has forced myself to give up naive reductionism but I believe that TGD allows generalization of Feynman diagram in such a manner that one gets rid of the infinities plaguing practically all existing theories. The purpose of this chapter is to develop general vision about how this might be achieved. The vision is based on generalization of mathematical structures discovered in the construction of topological quantum field theories (TQFT), conformal field theories (CQFT). In particular, the notions of Hopf algebras and quantum groups, and categories are central. The following gives a very concise summary of the basic ideas.

1. Feynman diagrams as generalized braid diagrams

The first key idea is that generalized Feynman diagrams with diagrams which are analogous to knot and link diagrams in the sense that diagrams involving loops are equivalent with tree diagrams. This would be a generalization of duality symmetry of string models.

TGD itself provides general arguments supporting same idea. The identification of absolute minimum of Kähler action or some other preferred extremal as a four-dimensional Feynman diagram characterizing particle reaction means that there is only single Feynman diagram instead of functional integral over 4-surfaces: this diagram is expected to be minimal one. S-matrix element as a representation of a path defining continuation of the configuration space spinor field between different sectors of it corresponding different 3-topologies leads also to the conclusion that all continuations and corresponding Feynman diagrams are equivalent. Universe as a computer metaphor idea allowing a quite concrete realization by generalization of what is meant by space-time point leads to the view that generalized Feynman diagrams characterize equivalent computations.

2. Coupling constant evolution from infinite number of critical values of Kähler coupling strength

The basic objection that this vision does not allow to understand coupling constant evolution involving loops in an essential manner can be circumvented. Quantum criticality requires that Kähler coupling strength α_K is analogous to critical temperature (so that the loops for configuration space integration vanish). The hypothesis motivated by the enormous vacuum degeneracy of Kähler action is that α_K has an infinite number of possible values labelled by p-adic length scales and also probably also by the dimensions of effective tensor factors associated with inclusions of von Neumann algebras known as hyper-finite type II_1 factors (so called Beraha numbers) as found already earlier. The dependence on p-adic length scale L_p corresponds to the usual renormalization group evolution whereas the latter dependence would correspond to angular resolution and finite-dimensional extensions of p-adic number fields R_p. Finite resolution and renormalization group evolution are forced by the algebraic continuation of rational number based physics to real and p-adic number fields since p-adic and real notions of distance between rational points differ dramatically.

3. R-matrices, complex numbers, quaternions, and octonions

A crucial observation is that physically equivalent R-matrices of 6-vertex models are labelled by points of CP_2 whereas maximal space of commuting R-matrices are labelled by points of 2-sphere S^2. CP_2 also labels maximal associative and thus quaternionic subspaces of 8-dimensional octonion space containing a preferred imaginary unit whereas S^2 labels the maximally commutative subspaces of quaternion space, which suggests that R-matrices and these structures correspond to each other.

Number theoretic vision leads to the notion of number theoretic compactification meaning that space-time surfaces could be regarded either as hyper-quaternionic, and thus maximal associative, 4-surfaces in M^8 regarded as the space of hyper-octonions or as surfaces in $M^4 \times CP_2$ (the imaginary units of hyper-octonions are multiplied with $\sqrt{-1}$ so that the number theoretical norm has Minkowski signature). No dynamics is involved and duality might be more appropriate term. Associativity constraint is an essential element of also Yang-Baxter equations.

These observations lead to a concrete proposal how how quantum classical correspondence is realized classically at space-time level. Each point of CP_2 corresponds to R-matrix and 3-surfaces can be identified as preferred sections of space-time surface by requiring that unitary R-matrices are commuting in the section (micro-causality) whereas commutativity fails for R-matrices corresponding to different values of time coordinate defined by this foliation.

4. Ordinary conformal symmetries act on the space of super-canonical conformal weights

TGD predicts two kinds of super-conformal symmetries.

a) The conformal symmetries defining super Kac-Moody representations are realized at light-like 3-surfaces appearing as boundaries of space-time sheets and boundaries between space-time regions with Euclidian and Minkowskian signature of metric. Conformal weights are half integer valued in this case.

b) Super-canonical conformal invariance acts at the level of imbedding space and corresponds to light-like 7-surfaces of form $X_l^3 \times CP_2 \subset M^4 \times CP_2$ believed to appear as causal determinants too. In this case conformal weights are complex numbers of form $\Delta = n/2 + iy$ and for physical states they are expected to reduce to the form $\Delta = 1/2 + iy$.

Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts.

The proposal is that the weights are mapped to the points of geodesic sphere of CP_2 (and thus also of space-time surface) labelling also mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. One can thus regard super-generators as conformal fields in space-time or complex plane in which conformal weights appear as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories induces by a pull-back a braid group action on the conformal labels of configuration space gamma matrices (super generators) and corresponding isometry generators.

The gamma matrices and isometry generators spanning the super-canonical algebra can be regarded as fields of a conformal field theory in the complex plane containing infinite number of punctures defined by the complex conformal weights. Quaternion conformal Kac Moody and Super Virasoro algebras act as symmetries of this theory and S-matrix of TGD should involve the n-point functions of this conformal field theory.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors having a close connection with non-trivial representations of braid group and quantum groups. The zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded a the infinite braid behind the von Neumann algebra. Contrary to the expectations, also trivial zeros seem to be important. Super-canonical algebra predicts that also the superposition for the imaginary parts of non-trivial zeros makes sense so that one would have an entire hierarchy of one-dimensional lattices depending on the number of non-trivial zeros included. The braids defined by sub-lattices defined by subsets of non-trivial zeros could be seen as completely integrable 1-dimensional spin chains leading to a hierarchy of quantum groups and braiding matrices naturally.

It seems that not only Riemann's zeta but also polyzetas could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to the zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically.

5. Equivalence of loop diagrams with tree diagrams from the axioms of generalized ribbon category

The fourth idea is that Hopf algebra related structures and appropriately generalized ribbon categories could provide a concrete realization of this picture. Generalized Feynman diagrams which are identified as braid diagrams with strands running in both directions of time and containing besides braid operations also boxes representing algebra morphisms with more than one incoming and outgoing strands describing particle reactions (3-particle vertex should be enough). In particular, fusion of 2-particles and decay of particle to two would correspond to generalizations of algebra product μ and co-product Δ to morphisms of the category defined by the super-canonical algebras associated with 3-surfaces with various topologies and conformal structures. The basic axioms for this structure generalizing Hopf algebra axioms state that diagrams with self energy loops, vertex corrections, and box diagrams are equivalent with tree diagrams.

6. Quantum criticality and renormalization group invariance

Quantum criticality means that renormalization group acts like isometry group at a fixed point rather than acting like a gauge symmetry as in the standard quantum field theory context. Despite this difference it is possible to understand how Feynman graph expansion with vanishing loop corrections relates to generalized Feynman graphs and a nice connection with the Hopf- and Lie algebra structures assigned by Connes and Kreimer to Feynman graphs emerges. The condition that loop diagrams are equivalent with tree diagrams gives explicit equations which might fix completely also the p-adic length scale evolution of vertices. Quantum criticality in principle fixes completely the values of the masses and coupling constants as a function of p-adic length scale.

7. The spectrum of zeros of Zeta and quantum TGD

The question whether the imaginary parts of the Riemann Zeta are linearly independent (as assumed in the previous work) or not is of crucial physical significance. Linear independence implies that the spectrum of the super-canonical weights is essentially an infinite-dimensional lattice. Otherwise a more complex structure results.

The hypothesis that p^{iy} is an algebraic phase for every prime implies that p^{iy} is expressible as a product of a Pythagorean phase and a root of unity for every prime p. The numerical evidence supporting the translational invariance of the correlations for the spectrum of zeros together with p-adic considerations leads to the working hypothesis that for any prime p one can express the spectrum of zeros as the product of a subset of Pythagorean prime phases and of a fixed subset U of roots of unity. The spectrum of zeros could be expressed as a union over the translates of the same basic spectrum defined by the roots of unity translated by the phase angles associated with a subset of Pythagorean phases: this is consistent with what the spectral correlations strongly suggest. That decompositions defined by different primes p yield the same spectrum would mean a powerful number theoretical symmetry realizing p-adicities at the level of the spectrum of Zeta.

The model for the scalar propagator as a partition function for the super-canonical algebra supports this view. The approximation that the zeros are linearly independent implies for any subalgebra defined by a finite set of zeros a universal spectrum of singularities for real values of mass squared, and at the limit of entire algebra the propagator is not mathematically defined since the singularities are dense in real axis. Linear independence however shifts the poles to complex values of mass and genuine resonances result and the propagator is expected to be well-defined for the entire algebra. The theory predicts universal momentum and p-adic length scale dependence of propagators in this picture and the option based on super-canonical algebra predicts very simple and elegant expression for the propagator with all the desired properties.

8. Quantum field theory formulation of quantum TGD

The super-canonical generalization of a 2-dimensional conformal field theory seems to be indispensable for the construction of S-matrix at the fundamental level and defines the vertices as n-point functions of a conformal field theory in turn used to construct S-matrix as tree diagrams. Also a quantum field theory defined for 3-surface X^3 belonging to the light like 7-surfaces $X_l^3 \times CP_2$ defining causal determinants seems to make sense. The QFT in question is determined by the absolute minimum $X^4(X^3)$ associated with the maximum of Kähler function and is consistent with the loop corrections in configuration space. The restriction to X^3 realizes quantum holography and means that a minimum amount of data about $X^4(X^3)$ is needed.

The modified Dirac action for the induced spinor fields at the maximum of the Kähler function with induced spinor fields identified as Grassmann algebra valued fields provides the sought for action. The fermionic propagator is defined by the inverse of the modified Dirac operator. Bosonic kinetic term is Grassmann algebra valued and vanishes for $\psi = 0$ and contributes nothing to the perturbation series in accordance with the effective freezing of the configuration space degrees of freedom. Since the fermionic action is free action, a divergence free quantum field theory is in question. One could also see the action as a fixed point of the map sending action to effective action in accordance with the idea that loop corrections vanish. Different sub-algebras of the super-canonical algebra correspond naturally to various conformally inequivalent configuration space metrics and infinite-dimensional subalgebras correspond to the singular limits when the space-time surface becomes vacuum extremal so that the modified Dirac operator vanishes identically and super-canonical propagator becomes ill-defined.

8.3.2 Was von Neumann right after all?

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix. This has turned out to be the case to the extend that a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.

1. Some background

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

2. How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

a) Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

b) Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

c) There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

d) Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of configuration space of 3-surfaces, etc....

3. Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

a) A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

b) At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

c) For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

d) The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

e) Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

8.3.3 Does TGD predict the spectrum of Planck constants?

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

1. Jones inclusions and quantization of Planck constant

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

a) The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

b) In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups. Mass spectrum is invariant under these scalings.

c) This predicts automatically arbitrarily large values of Planck constant and assigns the preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible in terms of square roots of rationals: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter correspond to these special values of Planck constant. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to re-scaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

d) These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_2n, E_6, E_8 are possible so that TGD seems to be able to mimick these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

2. The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant. The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 10 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 6 per cent in solar system to be compared with the accepted cosmological value of 4 per cent.

3. Identification of gravitational Planck constant as CP_2 Planck constant

$\hbar_{gr}$ can be interpreted as Planck constant associated with CP_2 degrees of freedom and its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

4. Large values of Planck constant and coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)} + 1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

8.4 PART IV: Applications

8.4.1 Cosmology and Astrophysics in Many-Sheeted Space-Time

This chapter is devoted to the applications of what might be called classical TGD to astrophysics and cosmology. In a well-defined sense classical TGD defined as the dynamics of space-time surface determining them as kind of generalized Bohr orbits can be regarded as an exact part of quantum theory and assuming quantum classical correspondence has served as an extremely valuable guideline in the attempts to interpret TGD, to form a view about what TGD really predicts, and to to guess what the underlying quantum theory could be and how it deviates from standard quantum theory. Also TGD inspired cosmology and astrophysics relies on this general picture.

1. Many-sheeted cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

The most important differences are following.

a) Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.

b) TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.

c) The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.

d) Dark matter hierarchy with dynamical quantized Planck constant implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.

e) Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

2. Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically. Concerning the understanding of cosmic strings a decisive breakthrough came through the identification of gravitational four-momentum as the difference of inertial momenta associated with matter and antimatter and the realization that the net inertial energy of the Universe vanishes. This forced to conclude cosmological constant in TGD Universe is non-vanishing. p-Adic length fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet. The recent value of the cosmological constant comes out correctly. The gravitational energy density described by the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and of magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

3. Dark matter and quantization of gravitational Planck constant

The notion of gravitational Planck constant having gigantic value is perhaps the most radical idea related to the astrophysical applications of TGD. D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

The earlier work with topological quantum computation [E9] had already led to the idea that Planck constant could depend on the quantum phase $q = exp(i\pi/n)$. The first attempts to understand the large values of the Planck constant led to a badly wrong formula for this dependence. The improved understanding of Jones inclusions and their role in TGD [C6] allowed to deduce an extremely simple formula for the Planck constant, as a matter for the two separate Planck constants assignable to with M^4 and CP_2 degrees of freedom appearing as scaling factors of the corresponding metrics. These Planck constants are given by the formulas $\hbar(M^4) = n(CP_2)\hbar_0$ and $\hbar(CP_2) = n(M^4)\hbar_0$ in terms of integers defining the corresponding quantum phases. The far reaching implication is that Planck constants can have arbitrarily large values.

Furthermore, the values of n for which the quantum phase is expressible using iterated square root operation (corresponding polygon is obtained by ruler and compass construction) are of special interest since they correspond to the lowest evolutionary levels for cognition so that corresponding systems should be especially abundant in the Universe. It should be noticed that this quantization does not depend at all on the parameter v_0 appearing in the formula of Nottale and this gives strong additional constraints to the ratios of planetary masses and also on the masses themselves if one assumes that the gravitational Planck constant corresponds to the values allowed by ruler and compass construction.

The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character. This would apply to QCD and to atoms with $Z > 137$ and to any other system.

8.4.2 Elementary particle vacuum functionals

Genus-generation correspondence is one of the basic ideas of TGD approach. In order to answer various questions concerning the plausibility of the idea, one should know something about the dependence of the elementary particle vacuum functionals on the vibrational degrees of freedom for the boundary component. The construction of the elementary particle vacuum functionals based on Diff invariance, 2-dimensional conformal symmetry, modular invariance plus natural stability requirements indeed leads to an essentially unique form of the vacuum functionals and one can understand why $g > 2$ bosonic families are experimentally absent and why lepton numbers are conserved separately.

An argument suggesting that the number of the light fermion families is three, is developed. The argument goes as follows. Elementary particle vacuum functionals represent bound states of g handles and vanish identically for hyper-elliptic surfaces having $g > 2$. Since all $g \leq 2$ surfaces are hyper-elliptic, $g \leq 2$ and $g > 2$ elementary particles cannot appear in same non-vanishing vertex and therefore decouple. The $g > 2$ vacuum functionals not vanishing for hyper-elliptic surfaces represent many particle states of $g \leq 2$ elementary particle states being thus unstable against the decay to $g \leq 2$ states. The failure of Z_2 conformal symmetry for $g > 2$ elementary particle vacuum functionals would in turn explain why they are heavy: this however not absolutely necessary since these particles would behave like dark matter in any case.

8.4.3 Massless states and particle massivation

The massless sector of the TGD and particle massivation is studied in this chapter. The identification of the spectrum of light particles reduces to two tasks: the construction of massless states and the identification of the states which remain light in p-adic thermodynamics.

1. Physical states as representations of super-canonical and Super Kac-Moody algebras

Physical states belong to the representation of super-canonical algebra and Super Kac-Moody algebra of $SO(2) \times E^2 \times SU(3) \times U(2)_{ew}$ associated with the 2-D surfaces X^2 defined by the intersections of 3-D light like causal determinants (CDs) with 7-D CDs $X^7 = X_l^3 \times CP_2$, where X_l^3 is boundary of future or past directed light cone. These 2-surfaces have interpretation as partons, and the effective 2-dimensionality means that the machinery of 2-D conformal field theories can be applied in the state construction.

The recipe is simple. Construct first a null state with a non-positive conformal weight using super-canonical generators, and then apply Super-Kac Moody generators to compensate this conformal weight to get a state with vanishing conformal weight and zero mass. Pose also the conditions that the commutator of super-canonical and super Kac-Moody algebras and corresponding commutator of Virasoro algebras annihilates physical states.

The conformal weights of super-canonical algebra generators are complex and in a well-defined sense expressible in terms of zeros of Riemann Zeta although the connection is much more subtle as thought originally [C1] and conformal weight cannot be regarded as quantum number in the standard sense of the word. More precisely, the arguments of [C1] suggest that radial conformal weight Δ for super-canonical algebra in fact depends on the point of geodesic sphere S^2 in CP_2 and is given in terms of the inverse $\zeta^{-1}(z)$ of Riemann ζ having the natural complex coordinate z of S^2 as argument. This implies a mapping of the radial conformal weights to the points of the geodesic sphere CP_2 serving in the role of "conformal heavenly sphere".

Linear combinations of zeros correspond to algebraic points in the intersections of real and p-adic partonic 2-surfaces and are thus in a unique role from the point of view of p-adicization. They can be also identified as conformal weights associated with parton as an n-particle state in the algebraic sense (these points correspond to arguments of n-point functions of conformal field theory in the construction of S-matrix). This discrete set of points defines in a natural manner number theoretic braid and a connection with braiding S-matrices emerges. This if one believes the basic conjecture that the numbers p^s, p prime and s zero of Riemann Zeta are algebraic numbers.

The original hypothesis was that the conformal weights of physical states are real. This would imply conformal confinement to which color confinement might reduce: what would happen that partons can have complex super-canonical conformal weights but particles have real conformal weights.

It has however turned out that there is actually no strong reason for the reality of the conformal weights.

The waves x^s with conformal weights $s = 1/2 + i\sum_k n_k y_k$, where $s_k = 1/2+iy_k$ is a zero of Zeta, define an orthogonal basis with respect to the inner product defined by the integration measure dx/x. These conformal weights can be assigned to both the eigenvalues of the modified Dirac operator and to radial logarithmic waves multiplying the Hamiltonians at $\delta M^4_\pm \times CP_2$ appearing in the construction of the configuration space geometry.

The imaginary part of the complex weight would allow to distinguish between particle and its phase conjugate (phase conjugate photons obey time reversed dynamics) by assigning to a particle inherent time orientation allowing to distinguish between positive energy particle propagating to the geometric future from a negative energy particle propagating to the geometric past. Obviously a strong correlation with second law of thermodynamics would emerge. It has turned out that the conformal weights $s = 1/2 + iy_k$ correspond to systems critical against transition changing the value of Planck constant.

One can also introduce the notion of bound state conformal weight in terms of Riemann polyzetas, and it turns out that number theoretic constraints imply that the net conformal weights in this case have the same spectrum as in single particle case and that irreducible n-particle bound states are possible only for $n = 2$ and 3. This suggests a connection with valence quark numbers of mesons and baryons [C1, E8] and perhaps also with family replication phenomenon (parton with genus $g = 0, 1, 2$ as conformally bound state of sphere and g handles so that only 3 stable particle families would result). The zeros of Zeta represent essentially non-stringy aspects of TGD being due to the fact that the basic objects are effectively 2-dimensional rather than 1-dimensional.

For spinor harmonics of CP_2 the correlation between color and electroweak quantum numbers is not correct. Super-canonical generators provide a natural mechanism allowing to cure the problem. Boson states are identified as bi-local bilinears of fermions and anti-fermions in X^2 characterized by charge matrices and conformally invariant correlation function. $BF\overline{F}$ coupling constants can be identified in terms of normalization factors of the boson states. The small value of gravitational coupling can be understood as resulting by a fractal mechanism reducing its value from the square of p-adic length L_p, and a concrete physical interpretation for the expression of gravitational constant in terms of CP_2 length derived from number theoretic arguments emerges. The presence of primes $2, 3, ...23, p$ in the expression of the gravitational constant can be interpreted in terms of multi-p p-adic fractality involving these primes.

If the primes $p = 2, 3, ...23$ are present, the question whether besides p-adic length scales $L_p \propto \sqrt{p}$ also their multiples $\sqrt{\prod_i q_i} L_p$, where $\{q_i\}$ forms a subset of $\{2, 3, ..., 23\}$ define fundamental length scales. The implication would be small-p p-adic fractality for these small primes with each p-adic length scale L_p taking the role of CP_2 length, and there indeed is some evidence for this kind of fractality.

2. Particle massivation

Particle massivation can be regarded as a change of the vanishing parton conformal weights describable as a thermal mixing with higher conformal weights. The observed mass squared is not p-adic thermal expectation of mass squared but that of conformal weight so that there are no problems with Lorentz invariance.

The space-time mechanism of massivation can be articulated in several manners.

a) CP_2 type vacuum extremals representing elementary particles have random light-like curve as an M^4 projection so that the average motion correspond to that of massive particle. Lighlike randomness gives rise to classical Virasoro conditions. p-Adic thermodynamics is consistent with this picture.

b) A possible candidate for the physical mechanism causing the thermal massivation is hydrodynamical mixing by the braiding flow. One can imagine several realizations for this flow. For instance a flow defined by the normal components of energy momentum tensor of the induced Kähler field at light like 3-D CDs describing the orbits of partons. Number theoretical approach leads to a purely number theoretical identification of braids and braiding flow and it seems that this flow might be more fundamental.

c) The fundamental parton level description of TGD is based on almost topological QFT for light-like 3-surfaces. Dynamics is constrained only by the requirement that CP_2 projection is for extremals of Chern-Simons action 2-dimensional and for off-shell states light-likeness is the only constraint. Hence a justification for the ergodic hydrodynamic flow as a fundamental cause of massivation emerges. The symmetries respecting light-likeness property correspond gives rise to Kac-Moody type algebra and super-canonical symmetries emerge also naturally as well as $N = 4$ character of super-conformal invariance. Four-momentum appears as non-conserved Noether charge (mass squared is however conserved) and has identification as gravitational four-momentum. Inertial momentum corresponds to the statistical average of gravitational four-momentum and p-adic thermodynamics is thus a natural description.

This mechanism cannot explain the massivation of electro-weak gauge bosons, which could be caused either by TGD variant of Higgs mechanism or by the fact that the charge matrices of W boson and left handed component of Z^0 are not covariantly constant, which together with the hydrodynamical mixing could lead to a loss of correlations. TGD indeed predicts a candidate for Higgs as a wormhole contact whose throats are identified as lightlike 3-surfaces and carry quantum numbers of fermion and antifermion and it is now clear that this is the correct option.

The underlying philosophy is that real number based TGD can be algebraically continued to various p-adic number fields. This gives justification for the use of p-adic thermodynamics although the mapping of p-adic thermal expectations to real counterparts is not completely unique. Instead of energy, the Super Kac-Moody Virasoro generator L_0 (essentially mass squared) is thermalized in p-adic thermodynamics. This guarantees Lorentz invariance. It is important to notice that four-momentum does not appear in the definition of super Virasoro generators. The reason is simply that four-momentum does not appear in the expression of super Virasoro generators as Noether charges associated with the modified Dirac action. The dependence of Virasoro generators on four-momentum would be in conflict with Lorentz invariance.

p-Adic thermodynamics forces to conclude that CP_2 radius is essentially the p-adic length scale $R \sim L$ and thus of order $R \simeq 10^4\sqrt{G}$ and therefore 10^4 times larger than the naive guess. Hence p-adic thermodynamics describes the mixing of states with vanishing conformal weights with their Super Kac-Moody Virasoro excitations having masses of order 10^{-4} Planck mass.

p-Adic temperature is quantized by purely number theoretical constraints (Boltzmann weight $exp(-E/kT)$ is replaced with p^{L_0/T_p}, $1/T_p$ integer) and fermions correspond to $T_p = 1$ whereas $T_p = 1/2$ seems to be the only reasonable choice for bosons. That mass squared, rather than energy, is a fundamental quantity at CP_2 length scale is also suggested by a simple dimensional argument (Planck mass squared is proportional to $\hbar$ so that it should correspond to a generator of some Lie-algebra (Virasoro generator L_0!)).

There is also modular contribution to the mass squared which can be estimated using elementary particle vacuum functionals in the conformal modular degrees of freedom of the partonic 2-surface. This contribution can be identified as a contribution coming from a thermodynamics in super-canonical Virasoro algebra which generates excitations of the ground states with negative conformal weight.

The predictions of the general theory are consistent with the earlier mass calculations, and the earlier ad hoc parameters disappear. In particular, optimal lowest order predictions for the charged lepton masses are obtained and photon, gluon and graviton appear as essentially massless particles. The negative conformal weight created by super-canonical generators can have arbitrarily large magnitude (ground state corresponds to a null state of super-conformal algebra annihilated by L_n, $n < 0$) so that an infinite hierarchy of exotic massless states is in principle possible. These states receive mass by the proposed mechanism and they are expected to be unstable but it remains to be shown that they do not appear in the spectrum of light particles. Since X^2 can have an arbitrarily large size and can even correspond to black hole horizon, the emergence of this complex structure of states is completely natural.

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